MIXING OF TWO LIQUIDS IN A DEVELOPED TURBULENCE REGION ACROSS A LOW-PENETRABILITY SEPARATOR

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An approximate solution is given to the problem of two liquids flowing one behind the other along a pipe under developed turbulence conditions and mixing across a low-penetrability separator installed between them.

In order to inhibit the mixing process in pipes, a special mechanical device can be installed to separate two liquids flowing one behind the other and to act as a separator which will prevent their mixing. Practical experience has shown, however, that absolute separation cannot be achieved with mechanical devices. Owing to its design characteristics and also owing to the wear of seals, a mechanical separator is always penetrable to some extent. This penetrability causes the liquids to leak across the separator in both directions and a mixture is thus produced in the zones before and behind the separator, the amount of this mixture depending on the magnitude of the leakage currents, on the hydrodynamic conditions of the main flow, and on the length of time the separator has been moving inside the mixing zone. As the separator moves, a pressure drop in the main flow developed by the braking action of the separator (Fig. 1) produces a forward leakage current (from zone II to zone I). A backward leakage current is produced by the braking action of the pipe walls along the boundary layer. The magnitudes of the leakage currents are important parameters of the mixing process with a separator in the contact zone between two liquids, and they in turn depend on the seal design and on the friction effects at this moving separator.

The development and the application of new separator designs must be based on quantitative data about these leakage currents, but neither a theoretical nor an experimental evaluation of the latter is possible. It is possible, however, to formulate the problem so that its solution can be used for evaluating the results of specially planned experiments and thus for pinpointing the effectiveness of one or another separator design.

We will assume that along the boundary between two incompressible liquids of low viscosity and similar densities (see Fig. 1) there is placed a single-barrier device (such separators include also, for example, spherical or solid-cylindrical ones).

In this case the mixing process will be simulated according to the Taylor hypothesis [1] based on the semiempirical equation of turbulent diffusion [2] under the following assumptions:

the presence of a mechanical separator has no significant effect on the average velocity distribution in the flow;

the mixture concentration in the developed turbulence region directly at the separator becomes with time almost uniform across the pipe section [3];

the mixture is "passive," i.e., does not influence the flow dynamics;

initially the separator lies in the plane of contact between the liquids flowing one behind the other.

With these assumptions, using a system of coordinates of which the origin moves with the plane of the separator, whilst the axis coincides with the pipe axis, one can represent the mixing process schematically by the following equations:

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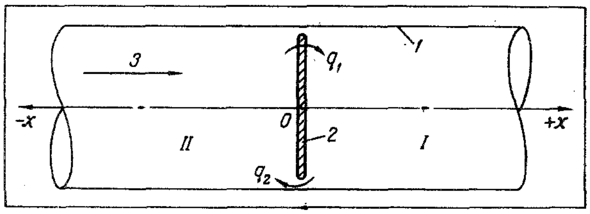

Fig. 1

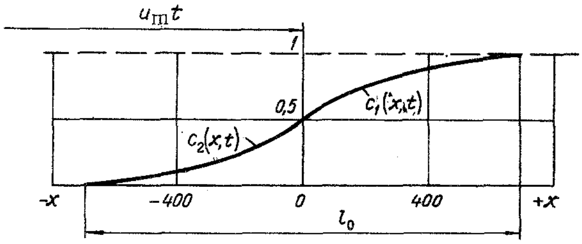

Fig. 2

Fig. 1. Schematic diagram showing the position of a separator in the pipeline: 1) pipe; 2) separator; 3) direction of flow; I) zone before the separator; II) zone behind the separator.

Fig. 2. Concentration of a displaced liquid along the mixing zone without a separator.

$$\frac{\partial c_1}{\partial t}+\frac{1}{f}\cdot\frac{\partial}{\partial x}\left[(q_1-q_2)c_1\right]-\frac{\partial}{\partial x}\left(K_1\frac{\partial c_1}{\partial x}\right)=0,\ x\in[0,\ \infty), \tag{1}$$

$$\frac{\partial c_2}{\partial t}+\frac{1}{f}\cdot\frac{\partial}{\partial x}\left[(q_1-q_2)c_2\right]-\frac{\partial}{\partial x}\left(K_2\frac{\partial c_2}{\partial x}\right)=0,\ x\in(-\infty,\ 0], \tag{2}$$

$$(q_1-q_2)\frac{1}{f}=u_{\mathrm{m}}-u_{\mathrm{s}}. \tag{3}$$

The solutions to Eqs. (1), (2), and (3) must satisfy the following initial and boundary conditions:

$$\begin{gathered} c_1(x,\ 0)=1 \quad \text{for } x\in[0,\ \infty),\\ c_2(x,\ 0)=0 \quad \text{for } x\in(-\infty,\ 0], \end{gathered} \tag{4}$$

$$\begin{gathered} \frac{q_1}{f}\left[c_2(0,\ t)-c_1(0,\ t)\right]+K_1\frac{\partial c_1(0,\ t)}{\partial x}=0,\\ \frac{q_2}{f}\left[c_1(0,\ t)-c_2(0,\ t)\right]-K_2\frac{\partial c_2(0,\ t)}{\partial x}=0,\\ \frac{\partial c_1(\infty,\ t)}{\partial x}=0, \quad \frac{\partial c_2(-\infty,\ t)}{\partial x}=0. \end{gathered} \tag{5}$$

The parameters q_1 and q_2 depend on many factors (on the seal geometry and penetrability, on the rate of wear, on the separator resistance forces, and on changes in the slope along the pipeline) and are generally unknown functions. We will consider the case of a separator provided with metallized elastic seals (wear resistant grades of steel are used for metallizing the seal elements) and moving inside a pipe with a constant slope, for which one may quite accurately assume that q_1 = const and q_2 = const. When one incompressible low-viscosity liquid flows behind another of similar density, it may be assumed that with time ($t \gg 0$) $K_1 = K_2 = K$.

The solution to the system of equations (1) and (2) can be expressed as

$$\begin{aligned} c_1(x,\ t)=1&-\frac{1}{2}\operatorname{erfc}\left[\frac{x-2\gamma_1 Kt}{2\sqrt{Kt}}\right]-\frac{\sqrt{u_1}-\sqrt{u_2}}{4\sqrt{u_2}}\\ &\times\exp\left[\frac{\sqrt{u_1}\left(\sqrt{u_1}-\sqrt{u_2}\right)x+t\left(\sqrt{u_1u_2}-u_1-u_2\right)}{K}\right]\\ &\times\operatorname{erfc}\left[\frac{x+\left(\sqrt{u_1}-\sqrt{u_2}\right)^2t}{2\sqrt{Kt}}\right]+\frac{\sqrt{u_1}+\sqrt{u_2}}{4\sqrt{u_2}}\\ &\times\exp\left[\frac{\sqrt{u_1}\left(\sqrt{u_1}+\sqrt{u_2}\right)x+t\sqrt{u_1u_2}\left(2\sqrt{u_1u_2}+u_1+u_2\right)}{K}\right]\operatorname{erfc}\left[\frac{x+\left(\sqrt{u_1}+\sqrt{u_2}\right)^2t}{2\sqrt{Kt}}\right]. \end{aligned} \tag{6}$$

$$\begin{aligned} c_2(x,\ t)=&\frac{1}{2}\operatorname{erfc}\left[\frac{-x+2\gamma_1 Kt}{2\sqrt{Kt}}\right]-\frac{\sqrt{u_1}-\sqrt{u_2}}{4\sqrt{u_1}}\\ &\times\exp\left[\frac{\sqrt{u_2}\left(\sqrt{u_1}-\sqrt{u_2}\right)x+t\sqrt{u_1u_2}\left(2\sqrt{u_1u_2}-u_1-u_2\right)}{K}\right] \end{aligned}$$

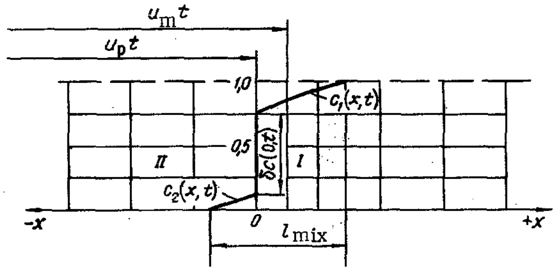

Fig. 3. Concentration of a displaced liquid along the moving zone of a mechanical separator when $q_1 > q_2$.

$$\times \operatorname{erfc}\left[\frac{-x+(\sqrt{u_1}-\sqrt{u_2})^2 t}{2\sqrt{Kt}}\right]-\frac{\sqrt{u_1}+\sqrt{u_2}}{4\sqrt{u_1}}$$

$$\times \exp\left[\frac{-\sqrt{u_2}(\sqrt{u_2}+\sqrt{u_1})\,x+t\sqrt{u_1u_2}\,(2\sqrt{u_1u_2}+u_1+u_2)}{K}\right]$$

$$\times \operatorname{erfc}\left[\frac{-x+(\sqrt{u_1}+\sqrt{u_2})^2 t}{2\sqrt{Kt}}\right]. \qquad (7)$$

We will begin to analyze this solution by considering the motion of two liquids one behind the other without a separator at their boundary. This case corresponds to the condition that

$$q_1 = q_2 = f u_m. \qquad (8)$$

The results of further analysis are presented in Figs. 2 and 3 (for $u_s = 1$ m/sec, $K = 4 \cdot 10^3$ cm^2/sec, $t = 10^5$ sec). The mixture concentration is shown distributed according to condition (8) in Fig. 2. The center of symmetry here, where $c_1(0, t) = c_2(0, t) = 0.5$, moves at a velocity u_s and the mixture is contained within the zone boundaries at l_0. The concentration distribution when a mechanical separator is present is shown in Fig. 3. This pattern corresponds to the condition that $q_1 > q_2$. In this case there is more mixture in zone I than in zone II. It becomes obvious from an analysis of Eqs. (6) and (7) that there will be more mixture in zone II when $q_1 < q_2$. No mixture is produced in zone I when $q_1 = 0$, and none is produced in zone II when $q_2 = 0$.

It also follows from Fig. 3 that the effectiveness of a mechanical separator can be adequately well characterized by the magnitude of the step change in concentration at the plane of the separator. Obviously, such a step change in the concentration is defined by the expression:

$$\delta c(0, t) = c_1(0, t) - c_2(0, t). \qquad (9)$$

A simultaneous consideration of Eqs. (6), (7), and (8) will make it evident that the step change in concentration due to the presence of a separator is maximum and equal to $\delta c(0, t) = 1$ when $q_1 = q_2 = 0$, and this corresponds to an absolutely impenetrable mechanical separator; no mixing of liquids occurs then in the contact zone, i.e., $l_{mix} = l_0$. When $q_1 \neq 0$ and $q_2 \neq 0$, on the other hand, then $\delta c(0, t) \to 0$ and $l_{mix} \to 0$ at $t \to \infty$. The latter indicates that the technical effectiveness of a penetrable mechanical separator decreases with time.

It must be noted, in conclusion, that the particular solution to the problem as shown here makes it possible to evaluate the technical effectiveness of single-barrier separators. With experimental data on $\delta c(0, t)$ available, this requires a solution of Eqs. (3) and (9) where f, t, u_m, u_s, K, and $\delta c(0, t)$ are given while q_1 and q_2 must be determined.

NOTATION

$c_1(x, t)$, $c_2(x, t)$	are the volumetric concentrations of the respective liquid displaced in zone I or zone II of the stream, m^3/m^3;
f	is the flow cross section in the pipe, m^2;
q_1, q_2	are the forward and backward leakage currents of the mixture, respectively, m^3/sec;
K_1, K_2	are the effective coefficients of turbulent diffusion, which characterizes the spreading of mixture in zones I and II, respectively, cm^2/sec;
u_m	is the mean linear flow velocity in the pipe, m/sec;
u_s	is the velocity of the mechanical separator, m/sec;
l_{mix}	is the length of stream segment containing the principal volume of mixture produced with a mechanical separator present, m;
l_0	is the length of stream segment within which the entire mixture is concentrated during overpumping in the absence of a mechanical separator, m;
t	is the time, sec;
0	is the origin of coordinates;

$\gamma_1 = (q_1 - q_2)/2Kf$;
$u_1 = q_1/f$;
$u_2 = q_2/f$;

$$\operatorname{erfc} z = 1 - \frac{2}{\sqrt{\pi}} \int_0^z \exp(-z^2)dz;$$

x is the longitudinal coordinate, m.

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